Quantum Correlations are Weaved by the Spinors of the Euclidean Primitives

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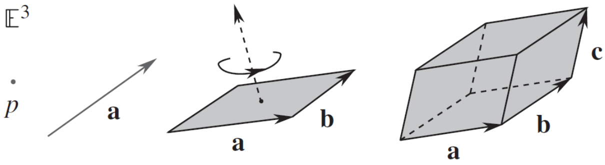
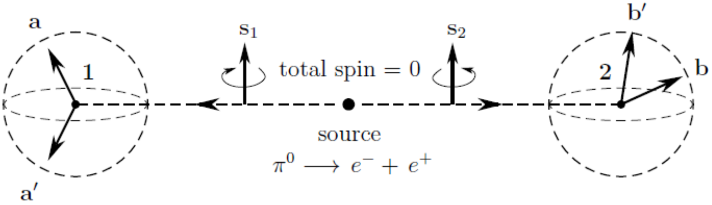


AGACSE 2021 – Conference in Honor of Prof. David Hestenes

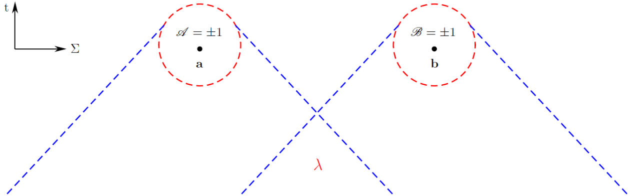
07 September 2021 – Brno, Czech Republic

Refs: DOI: 10.1098/rsos.180526, *Royal Society Open Science*
DOI: 10.1109/ACCESS.2019.2941275, *IEEE Access*
DOI: 10.1109/ACCESS.2020.3031734, *IEEE Access*
DOI: 10.1109/ACCESS.2021.3076449, *IEEE Access*
DOI: 10.1007/s10773-014-2412-2, *Int. J. Theor. Phys.*

The EPR-Bohm or Bell-test Experiment



Measurements of spin components on each separated fermion are performed by Alice and Bob at remote stations 1 and 2, providing binary outcomes $+1$ or -1 along freely chosen directions **a** and **b**.



The common cause λ is predetermined in the overlap of backward light-cones of Alice and Bob, encoding their shareable information.

The Proposed Hypothesis (2007)

The quantum correlations we observe in Nature can be understood as correlations among the limiting scalar points of an octonion-like 7-sphere, which is an algebraic representation space of quaternionic 3-sphere. One can define a 3-sphere as the set of unit quaternions:

$$S^3 := \left\{ \mathbf{q}(\psi,\, \mathbf{r}) := \exp\left[\mathbf{J}(\mathbf{r})\, \frac{\psi}{2} \right] \,\middle|\, ||\,\mathbf{q}\,(\psi,\, \mathbf{r})\,|| = \varrho_r \right\}.$$

Here $\mathbf{J}(\mathbf{r})$ is a bivector (or pure quaternion) rotating about a vector $\mathbf{r} \in \mathrm{I\!R}^3$, with rotation angle $0 \leq \psi < 4\pi$, and ϱ_r is the radius of S^3.

Thus, the strong correlations we observe in Bell-test experiments can be understood local-realistically if the 3D physical space, $\mathbb{E}^3$, is modelled as a closed and compact quaternionic 3-sphere using Geometric Algebra, instead of open space $\mathrm{I\!R}^3$ using "vector algebra."

Tsirel'son's Bounds are a Consequence of this Hypothesis:

$$-2\sqrt{2} \leqslant \mathcal{E}(\mathbf{a},\, \mathbf{b}) + \mathcal{E}(\mathbf{a},\, \mathbf{b}') + \mathcal{E}(\mathbf{a}',\, \mathbf{b}) - \mathcal{E}(\mathbf{a}',\, \mathbf{b}') \leqslant 2\sqrt{2}.$$

Friedmann-Lemaître-Robertson-Walker Spacetime

The above is by no means an *ad hoc* hypothesis. S^3 happens to be isomorphic to the spatial part of a well known solution of Einstein's field equations of general relativity, representing a *closed* Universe with a constant positive spatial curvature via the FRW line element

$$ds^2 = dt^2 - a^2(t)\, d\boldsymbol{\Sigma}^2, \quad d\boldsymbol{\Sigma}^2 = \left[\frac{d\rho^2}{1-\kappa\,\rho^2} + \rho^2 d\boldsymbol{\Omega}^2\right].$$

Here $a(t)$ is the scale factor, $\boldsymbol{\Sigma}$ is a spacelike hypersurface, ρ is the radial coordinate within $\boldsymbol{\Sigma}$, κ is the "normalized" curvature of $\boldsymbol{\Sigma}$, and $\boldsymbol{\Omega}$ is a solid angle within $\boldsymbol{\Sigma}$. For terrestrial scenarios, $a(t) = 1$.

The above line element permits three possible geometries with the product topology $\mathrm{I\!R} \times \boldsymbol{\Sigma}$. The hypersurfaces $\boldsymbol{\Sigma}$ can be isomorphic to $\mathrm{I\!R}^3$, S^3, or H^3. But only S^3 represents a closed universe with a positive curvature. Moreover, the cosmic microwave background spectra now prefers the positive curvature at 99% confidence level.

The Special Theorem — Correlations within S^3

The quantum mechanical correlations predicted by the entangled singlet state can be deterministically understood as classical, local, and realistic correlations among the pairs of limiting scalar points with values ± 1 of a quaternionic 3-sphere, defined by the functions

$$S^3 \ni \mathscr{A}(\mathbf{a},\,\lambda^k) := \lim_{\mathbf{s}_1 \to \mathbf{a}} \{+\,\mathbf{q}(\eta_{\mathbf{a}\mathbf{s}_1},\,\mathbf{r}_1)\} \equiv \lim_{\mathbf{s}_1 \to \mathbf{a}} \left\{-\,\mathbf{D}(\mathbf{a})\,\mathbf{L}(\mathbf{s}_1,\,\lambda^k)\right\}$$

$$\xrightarrow[\mathbf{s}_1 \to \mathbf{a}]{} \begin{Bmatrix} +1 & \text{if} & \lambda^k = +1 \\ -1 & \text{if} & \lambda^k = -1 \end{Bmatrix}, \text{ with } \left\langle \mathscr{A}(\mathbf{a},\,\lambda^k) \right\rangle = 0, \text{ and}$$

$$S^3 \ni \mathscr{B}(\mathbf{b},\,\lambda^k) := \lim_{\mathbf{s}_2 \to \mathbf{b}} \{-\,\mathbf{q}(\eta_{\mathbf{s}_2\mathbf{b}},\,\mathbf{r}_2)\} \equiv \lim_{\mathbf{s}_2 \to \mathbf{b}} \left\{+\,\mathbf{L}(\mathbf{s}_2,\,\lambda^k)\,\mathbf{D}(\mathbf{b})\right\}$$

$$\xrightarrow[\mathbf{s}_2 \to \mathbf{b}]{} \begin{Bmatrix} -1 & \text{if} & \lambda^k = +1 \\ +1 & \text{if} & \lambda^k = -1 \end{Bmatrix}, \text{ with } \left\langle \mathscr{B}(\mathbf{b},\,\lambda^k) \right\rangle = 0,$$

where the bivectors $\mathbf{L}(\mathbf{s}_1,\,\lambda^k)$ and $\mathbf{L}(\mathbf{s}_2,\,\lambda^k)$ represent the fermions emerging from a source that are detected by two detector bivectors $\mathbf{D}(\mathbf{a})$ and $\mathbf{D}(\mathbf{b})$, and $\lambda = \pm$ is the orientation of S^3 relating $\mathbf{L}$ to $\mathbf{D}$:

$$\mathbf{L}(\mathbf{n},\,\lambda) = \lambda\,\mathbf{D}(\mathbf{n}) \iff \mathbf{D}(\mathbf{n}) = \lambda\,\mathbf{L}(\mathbf{n},\,\lambda)\,.$$

A Simple Proof of the Special Theorem

What will be the value of the product $\mathscr{A}\mathscr{B}$ *within* S^3 when the results $\mathscr{A}$ and $\mathscr{B}$ are observed by Alice and Bob simultaneously?

For $\mathbf{s}_1 = \mathbf{s}_2$ the product $\mathscr{A}\mathscr{B}$ of measurement results reduces to

$$S^3 \ni \mathscr{A}(\mathbf{a},\,\lambda^k)\,\mathscr{B}(\mathbf{b},\,\lambda^k) \longrightarrow \lim_{\substack{\mathbf{s}_1 \to \mathbf{a} \\ \mathbf{s}_2 \to \mathbf{b}}} \left\{-\,\mathbf{q}(\eta_{\mathbf{ab}},\,\mathbf{r}_0)\right\}$$

$$= \lim_{\substack{\mathbf{s}_1 \to \mathbf{a} \\ \mathbf{s}_2 \to \mathbf{b}}} \left\{-\cos(\,\eta_{\mathbf{ab}}) - \mathbf{L}(\mathbf{r}_0,\,\lambda^k)\,\sin(\,\eta_{\mathbf{ab}})\right\},$$

with $\quad \mathbf{r}_0 = \dfrac{(\mathbf{a}\cdot\mathbf{s}_1)(\mathbf{s}_2\times\mathbf{b})+(\mathbf{s}_2\cdot\mathbf{b})(\mathbf{a}\times\mathbf{s}_1)-(\mathbf{a}\times\mathbf{s}_1)\times(\mathbf{s}_2\times\mathbf{b})}{\sin\,(\,\eta_{\mathbf{ab}})}$

so that $\quad \lim\limits_{\substack{\mathbf{s}_1 \to \mathbf{a} \\ \mathbf{s}_2 \to \mathbf{b}}} \mathbf{r}_0 = \vec{\mathbf{0}}\,,$

giving $\quad \mathcal{E}^{\rm Bell}_{L.R.}(\mathbf{a},\,\mathbf{b}) = \lim\limits_{n\gg 1}\left[\dfrac{1}{n}\sum\limits_{k=1}^{n}\mathscr{A}(\mathbf{a},\,\lambda^k)\,\mathscr{B}(\mathbf{b},\,\lambda^k)\right]$

$$= \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{\substack{\mathbf{s}_1 \to \mathbf{a} \\ \mathbf{s}_2 \to \mathbf{b}}}\left\{-\cos(\,\eta_{\mathbf{ab}}) - \mathbf{L}(\mathbf{r}_0,\,\lambda^k)\,\sin(\,\eta_{\mathbf{ab}})\right\}\right]$$

$$= -\cos(\,\eta_{\mathbf{ab}}) - \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\mathbf{L}(\vec{\mathbf{0}},\,\lambda^k)\,\sin(\,\eta_{\mathbf{ab}})\right] = -\cos(\,\eta_{\mathbf{ab}}).$$

Quantum Prediction

Orientation λ of S^3 is of Only Relative Significance

The spins $\mathbf{L}(\mathbf{s}_1,\,\lambda)$ and $\mathbf{L}(\mathbf{s}_2,\,\lambda)$ and the detectors $\mathbf{D}(\mathbf{a})$ and $\mathbf{D}(\mathbf{b})$ are two entirely unrelated physical systems. Alice and Bob have no knowledge of the handedness of the spins until their measurements. Therefore, spins and detectors are represented by different bases:

$$L_i(\lambda)\,L_j(\lambda) \,=\, -\,\delta_{ij} \,-\, \epsilon_{ijk}\,L_k(\lambda)$$

and

$$D_i\,D_j \,=\, -\,\delta_{ij} \,-\, \epsilon_{ijk}\,D_k$$

These bases are related *only* by the orientation λ of the 3-sphere:

$$L_i(\lambda) \,=\, \lambda\,D_i \;\;\Longleftrightarrow\;\; D_i \,=\, \lambda\,L_i(\lambda)\,.$$

The handedness relation between the two bivector bases is therefore

$$L_1(\lambda)L_2(\lambda)L_3(\lambda) = \lambda\,D_1\,D_2\,D_3 = \pm\,D_1\,D_2\,D_3.$$

Consequently, the *perspectives* of spins and detectors are related as

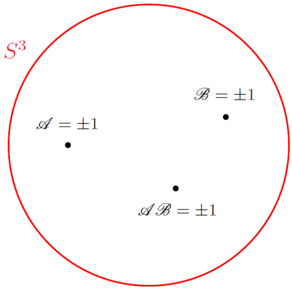
$$\mathbf{L}(\mathbf{a},\,\lambda=+1)\;\mathbf{L}(\mathbf{b},\,\lambda=+1) = \mathbf{D}(\mathbf{a})\;\mathbf{D}(\mathbf{b}),$$

but

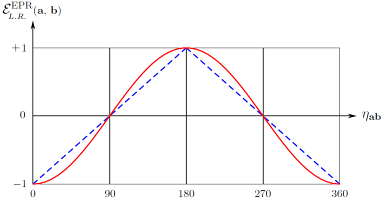
$$\mathbf{L}(\mathbf{a},\,\lambda=-1)\;\mathbf{L}(\mathbf{b},\,\lambda=-1) = \mathbf{D}(\mathbf{b})\;\mathbf{D}(\mathbf{a}).$$

Several people have *independently* verified the above theorem using a variety of softwares such as *Python*, *Mathematica*, *R*, and *Maple*.

The Special Theorem in Pictures



The results $\mathscr{A}$ and $\mathscr{B}$ are events in S^3. Since S^3 remains closed under multiplication, their product $\mathscr{A}\mathscr{B} = \pm 1$ also remains in S^3.



The cosine curve depicts the local-realistic correlations predicted within S^3 and the dotted lines depict those predicted within $\mathrm{I\!R}^3$.

8D Even Sub-algebra $\mathcal{K}^\lambda$ of the 16D Algebra $\text{Cl}_{4,0}$

$$\begin{array}{ccc}
\mathbb{R}^8 \simeq \text{Cl}_{3,0} & \xrightarrow{\;\cup\{\mathbf{e}_\infty\}\;} & \mathcal{K}^\lambda \hookleftarrow S^7 \\
\uparrow & & \uparrow \\
\text{Cl}_{3,0} \supset \mathbb{R}^3 & \xrightarrow{\;\cup\{\infty\}\;} & \mathbb{R}^4 \hookleftarrow S^3
\end{array}$$

Quaternionic 3-sphere is sufficient for understanding the singlet correlations local-realistically. But it is not sufficient for more general quantum correlations. What is needed is an algebraic representation space of S^3. To find that, let us recall that the algebraic representation space of $\mathbb{R}^3$ is the Geometric Algebra

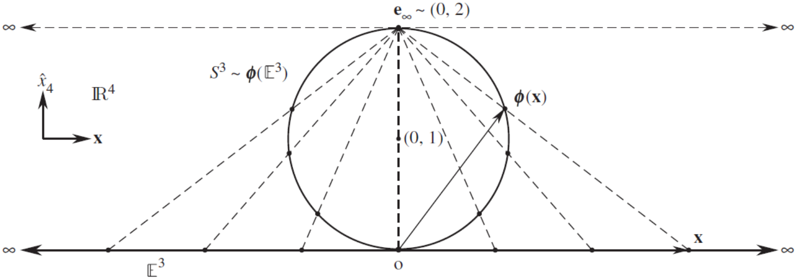
$$\text{Cl}_{3,0} = \text{span}\{\, 1,\; \mathbf{e}_x,\; \mathbf{e}_y,\; \mathbf{e}_z,\; \mathbf{e}_x\mathbf{e}_y,\; \mathbf{e}_z\mathbf{e}_x,\; \mathbf{e}_y\mathbf{e}_z,\; \mathbf{e}_x\mathbf{e}_y\mathbf{e}_z =: I_3 \,\}$$

by the bijection $\mathcal{F} : \mathbb{R}^3 = \text{span}\{\, \mathbf{e}_x,\, \mathbf{e}_y,\, \mathbf{e}_z\} \rightarrow \mathbb{R}^8 \approx \text{Cl}_{3,0}$. And

$$S^3 = \mathbb{R}^3 \cup \{\infty\} \quad \longleftarrow \text{ one-point compactification of } \mathbb{R}^3.$$

S^3 can be constructed by adding a single point to $\mathbb{R}^3$ at infinity.

"One-point Compactification" of $\mathrm{Cl}_{3,0}$ leads to $\mathcal{K}^\lambda$



$$\mathbf{e}_x\mathbf{e}_y\mathbf{e}_z \equiv I_3 \xrightarrow{\;\mathbf{e}_\infty\;} I_3\mathbf{e}_\infty \equiv \mathbf{e}_x\mathbf{e}_y\mathbf{e}_z\mathbf{e}_\infty$$

$$\mathcal{K}^\lambda = \mathrm{span}\{1,\ \lambda\mathbf{e}_x\mathbf{e}_y,\ \lambda\mathbf{e}_z\mathbf{e}_x,\ \lambda\mathbf{e}_y\mathbf{e}_z,\ \lambda\mathbf{e}_x\mathbf{e}_\infty,\ \lambda\mathbf{e}_y\mathbf{e}_\infty,\ \lambda\mathbf{e}_z\mathbf{e}_\infty,\ \lambda I_3\mathbf{e}_\infty\}$$

↑ $\mathbf{e}_\infty$ gives the even subalgebra $\mathcal{K}^\lambda$ of $\mathrm{Cl}_{4,0}$

$$\mathrm{Cl}_{3,0} = \mathrm{span}\{1,\ \lambda\mathbf{e}_x,\ \lambda\mathbf{e}_y,\ \lambda\mathbf{e}_z,\ \lambda\mathbf{e}_x\mathbf{e}_y,\ \lambda\mathbf{e}_z\mathbf{e}_x,\ \lambda\mathbf{e}_y\mathbf{e}_z,\ \lambda\mathbf{e}_x\mathbf{e}_y\mathbf{e}_z =: \lambda I_3\}$$

Relationships Among Various Algebras and Spaces

$\mathbb{R}^4 \hookleftarrow S^3$ $\xrightarrow[\|\mathbb{Q}_{z1}\mathbb{Q}_{z2}\| = \|\mathbb{Q}_{z1}\|\,\|\mathbb{Q}_{z2}\| \implies \text{Division Algebra}]{\mathcal{K}^\lambda = \left\{ \mathbb{Q}_z = \mathbf{q}_r + \mathbf{q}_d\,\varepsilon \;\middle|\; \mathbf{q}_r \perp \mathbf{q}_d \iff \|\mathbb{Q}_z\| = \text{a scalar} \right\}}$ $S^7 \hookrightarrow \mathbb{R}^8$

Left vertical arrow ($\mathbb{R}^4 \to S^3$):

$\mathbb{R}^4 \to \mathbb{R}^3 \cup \{\infty\}$

$\Updownarrow$

$\|\mathbf{q}_r\| = \varrho_r$

$\uparrow$

Quaternions

Right vertical arrow ($\mathcal{K}^\lambda \to S^7$):

$\mathbb{R}^8 \to \mathbb{R}^7 \cup \{\infty\}$

$\Updownarrow$

$\|\mathbb{Q}_z\| = \sqrt{\varrho_r^2 + \varrho_d^2}$

$\uparrow$

$\mathbb{Q}_z = \mathbf{q}_r + \mathbf{q}_d\,\varepsilon$

$\mathbb{R}^8 \simeq \mathrm{Cl}_{3,0} \supset \mathbb{R}^4$ $\xrightarrow[\mathcal{K}^\lambda \simeq \mathbb{R}^8 \text{ is an 8D normed but not yet a division algebra}]{\text{4 even basis vectors} \cup \text{those} \times I_3^\dagger \mathbf{e}_\infty = \text{8 even basis vectors}}$ $\mathcal{K}^\lambda \subset \mathrm{Cl}_{4,0} \simeq \mathbb{R}^{16}$

Left vertical arrow ($\mathrm{Cl}_{3,0} \supset \mathbb{R}^3 \to \mathbb{R}^4$):

Even subalgebra of $\mathrm{Cl}_{3,0}$

$\uparrow$

Select only the 4 even basis vectors from 2^3 basis vectors of $\mathrm{Cl}_{3,0}$

Right vertical arrow ($\mathrm{Cl}_{3,0} \to \mathcal{K}^\lambda$):

Even subalgebra of $\mathrm{Cl}_{4,0}$

$\uparrow$

Take 4 even basis vectors $\cup$ 4 odd basis vectors $\times\, \mathbf{e}_\infty$ from $2^4 = 16$ basis vectors

$\mathrm{Cl}_{3,0} \supset \mathbb{R}^3$ $\xrightarrow[\mathrm{Cl}_{3,0} \text{ is the algebra of orthogonal directions in } \mathbb{R}^3]{\text{The basis vectors } \mathbf{e}_x,\, \mathbf{e}_y,\, \mathbf{e}_z \text{ of } \mathbb{R}^3 \cup \text{their products}}$ $\mathrm{Cl}_{3,0} \simeq \mathbb{R}^8$

$$\mathcal{K}^\lambda = \mathrm{span}\{1,\ \lambda\mathbf{e}_x\mathbf{e}_y,\ \lambda\mathbf{e}_z\mathbf{e}_x,\ \lambda\mathbf{e}_y\mathbf{e}_z,\ \lambda\mathbf{e}_x\mathbf{e}_\infty,\ \lambda\mathbf{e}_y\mathbf{e}_\infty,\ \lambda\mathbf{e}_z\mathbf{e}_\infty,\ \lambda I_3\mathbf{e}_\infty\}$$

Multiplication Table of $\mathcal{K}^\lambda$

$*$	1	$\lambda\,\mathbf{e}_x\mathbf{e}_y$	$\lambda\,\mathbf{e}_z\mathbf{e}_x$	$\lambda\,\mathbf{e}_y\mathbf{e}_z$	$\lambda\,\mathbf{e}_x\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_y\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_z\mathbf{e}_\infty$	$\lambda\,I_3\mathbf{e}_\infty$
1	1	$\lambda\,\mathbf{e}_x\mathbf{e}_y$	$\lambda\,\mathbf{e}_z\mathbf{e}_x$	$\lambda\,\mathbf{e}_y\mathbf{e}_z$	$\lambda\,\mathbf{e}_x\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_y\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_z\mathbf{e}_\infty$	$\lambda\,I_3\mathbf{e}_\infty$
$\lambda\,\mathbf{e}_x\mathbf{e}_y$	$\lambda\,\mathbf{e}_x\mathbf{e}_y$	-1	$\mathbf{e}_y\mathbf{e}_z$	$-\mathbf{e}_z\mathbf{e}_x$	$-\mathbf{e}_y\mathbf{e}_\infty$	$\mathbf{e}_x\mathbf{e}_\infty$	$I_3\mathbf{e}_\infty$	$-\mathbf{e}_z\mathbf{e}_\infty$
$\lambda\,\mathbf{e}_z\mathbf{e}_x$	$\lambda\,\mathbf{e}_z\mathbf{e}_x$	$-\mathbf{e}_y\mathbf{e}_z$	-1	$\mathbf{e}_x\mathbf{e}_y$	$\mathbf{e}_z\mathbf{e}_\infty$	$I_3\mathbf{e}_\infty$	$-\mathbf{e}_x\mathbf{e}_\infty$	$-\mathbf{e}_y\mathbf{e}_\infty$
$\lambda\,\mathbf{e}_y\mathbf{e}_z$	$\lambda\,\mathbf{e}_y\mathbf{e}_z$	$\mathbf{e}_z\mathbf{e}_x$	$-\mathbf{e}_x\mathbf{e}_y$	-1	$I_3\mathbf{e}_\infty$	$-\mathbf{e}_z\mathbf{e}_\infty$	$\mathbf{e}_y\mathbf{e}_\infty$	$-\mathbf{e}_x\mathbf{e}_\infty$
$\lambda\,\mathbf{e}_x\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_x\mathbf{e}_\infty$	$\mathbf{e}_y\mathbf{e}_\infty$	$-\mathbf{e}_z\mathbf{e}_\infty$	$I_3\mathbf{e}_\infty$	-1	$-\mathbf{e}_x\mathbf{e}_y$	$\mathbf{e}_z\mathbf{e}_x$	$-\mathbf{e}_y\mathbf{e}_z$
$\lambda\,\mathbf{e}_y\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_y\mathbf{e}_\infty$	$-\mathbf{e}_x\mathbf{e}_\infty$	$I_3\mathbf{e}_\infty$	$\mathbf{e}_z\mathbf{e}_\infty$	$\mathbf{e}_x\mathbf{e}_y$	-1	$-\mathbf{e}_y\mathbf{e}_z$	$-\mathbf{e}_z\mathbf{e}_x$
$\lambda\,\mathbf{e}_z\mathbf{e}_\infty$	$\lambda\,\mathbf{e}_z\mathbf{e}_\infty$	$I_3\mathbf{e}_\infty$	$\mathbf{e}_x\mathbf{e}_\infty$	$-\mathbf{e}_y\mathbf{e}_\infty$	$-\mathbf{e}_z\mathbf{e}_x$	$\mathbf{e}_y\mathbf{e}_z$	-1	$-\mathbf{e}_x\mathbf{e}_y$
$\lambda\,I_3\mathbf{e}_\infty$	$\lambda\,I_3\mathbf{e}_\infty$	$-\mathbf{e}_z\mathbf{e}_\infty$	$-\mathbf{e}_y\mathbf{e}_\infty$	$-\mathbf{e}_x\mathbf{e}_\infty$	$-\mathbf{e}_y\mathbf{e}_z$	$-\mathbf{e}_z\mathbf{e}_x$	$-\mathbf{e}_x\mathbf{e}_y$	1

Products $\mathbb{Q}_z\mathbb{Q}_z^\dagger$ Resemble Split Complex Numbers

Consider an arbitrary multivector in $\mathcal{K}^\lambda$, such as

$$\mathbb{Q}_z = q_0 + q_1\,\lambda\mathbf{e}_x\mathbf{e}_y + q_2\,\lambda\mathbf{e}_z\mathbf{e}_x + q_3\,\lambda\mathbf{e}_y\mathbf{e}_z + q_4\,\lambda\mathbf{e}_x\mathbf{e}_\infty + q_5\,\lambda\mathbf{e}_y\mathbf{e}_\infty + q_6\,\lambda\mathbf{e}_z\mathbf{e}_\infty + q_7\,\lambda I_3\mathbf{e}_\infty.$$

It can be written more conveniently in terms of two quaternions as

$$\boxed{\mathbb{Q}_z = \mathbf{q}_r + \mathbf{q}_d\,\varepsilon}, \quad \text{where}$$

$$\mathbf{q}_r := q_0 + q_1\,\lambda\,\mathbf{e}_x\mathbf{e}_y + q_2\,\lambda\,\mathbf{e}_z\mathbf{e}_x + q_3\,\lambda\,\mathbf{e}_y\mathbf{e}_z,\ \ ||\mathbf{q}_r|| = \sqrt{\mathbf{q}_r\mathbf{q}_r^\dagger} = \varrho_r,$$

$$\mathbf{q}_d := -q_7 + q_6\,\mathbf{e}_x\mathbf{e}_y + q_5\,\mathbf{e}_z\mathbf{e}_x + q_4\,\mathbf{e}_y\mathbf{e}_z,\ \ ||\mathbf{q}_d|| = \sqrt{\mathbf{q}_d\mathbf{q}_d^\dagger} = \varrho_d,$$

and $\boxed{\varepsilon := -\lambda I_3\mathbf{e}_\infty}$ is a pseudoscalar satisfying $\varepsilon^2 = +1$ and $\varepsilon^\dagger = \varepsilon$, so that

$$\begin{aligned}
\mathbb{Q}_z\mathbb{Q}_z^\dagger &= \left(\mathbf{q}_r\,\mathbf{q}_r^\dagger + \mathbf{q}_d\,\mathbf{q}_d^\dagger\right) + \left(\mathbf{q}_r\,\mathbf{q}_d^\dagger + \mathbf{q}_d\,\mathbf{q}_r^\dagger\right)\varepsilon \\
&= \left(\varrho_r^2 + \varrho_d^2\right) + \left(-2\,q_0q_7 + 2\,\lambda\,q_1q_6 + 2\,\lambda\,q_2q_5 + 2\,\lambda\,q_3q_4\right)\,\varepsilon \\
&= (\text{a scalar}) + (\text{a scalar})\,\varepsilon \ \longleftarrow \text{like a split complex number} \\
&= (\text{a scalar}) + (\text{a pseudoscalar}).
\end{aligned}$$

Composition Law of Norms Holds for All $\mathbb{Q}_z$ in $\mathcal{K}^\lambda$

In Appendix B of arxiv.org/abs/1908.06172 I have proved that, given two multivectors, $\mathbb{Q}_{z1}$ and $\mathbb{Q}_{z2}$, in $\mathcal{K}^\lambda$, the product of their norms will always satisfy the norm relation, or composition law

$$\boxed{||\mathbb{Q}_{z1}\mathbb{Q}_{z2}||^2 \;=\; ||\mathbb{Q}_{z1}||^2 \; ||\mathbb{Q}_{z2}||^2},$$

just as $\mathbf{q}_r$ and $\mathbf{q}_d$ themselves do, where the norm of each $\mathbb{Q}_z \in \mathcal{K}^\lambda$ is defined as $\boxed{||\mathbb{Q}_z|| := \sqrt{\mathbb{Q}_z\mathbb{Q}_z^\dagger}}$, which remains positive definite:

$$\boxed{||\mathbb{Q}_z|| = 0 \iff \mathbb{Q}_z = 0}.$$

Both sides of the above composition law work out to be equal to

$$\left\{\left(\varrho_{r1}^2 + \varrho_{d1}^2\right)\left(\varrho_{r2}^2 + \varrho_{d2}^2\right) + \left(\mathbf{q}_{r1}\,\mathbf{q}_{d1}^\dagger + \mathbf{q}_{d1}\,\mathbf{q}_{r1}^\dagger\right)\left(\mathbf{q}_{r2}\,\mathbf{q}_{d2}^\dagger + \mathbf{q}_{d2}\,\mathbf{q}_{r2}^\dagger\right)\right\}$$
$$+\left\{\left(\varrho_{r1}^2 + \varrho_{d1}^2\right)\left(\mathbf{q}_{r2}\,\mathbf{q}_{d2}^\dagger + \mathbf{q}_{d2}\,\mathbf{q}_{r2}^\dagger\right) + \left(\varrho_{r2}^2 + \varrho_{d2}^2\right)\left(\mathbf{q}_{r1}\,\mathbf{q}_{d1}^\dagger + \mathbf{q}_{d1}\,\mathbf{q}_{r1}^\dagger\right)\right\}\varepsilon.$$

This too resembles a split complex number and its squareroot gives

$$\boxed{||\mathbb{Q}_{z1}\,\mathbb{Q}_{z2}|| = ||\mathbb{Q}_{z1}||\,||\mathbb{Q}_{z2}||}.$$

Inconsistency in the Alleged Counterexample

Let us consider the following two multivectors in the algebra $\mathcal{K}^\lambda$:

$$X = \frac{1}{\sqrt{2}}(1+\varepsilon) \neq 0 \quad \text{and} \quad Y = \frac{1}{\sqrt{2}}(1-\varepsilon) \neq 0,$$

where $\varepsilon^\dagger = \varepsilon$ and $\varepsilon^2 = 1$. If we use scalar products $Z \cdot Z^\dagger = ||Z||^2$ to evaluate the norms $||X||$ and $||Y||$, then we get nonzero values

$$||X|| = \left|\left|\frac{1}{\sqrt{2}}(1+\varepsilon)\right|\right| = \sqrt{\frac{1}{2}(1+\varepsilon)\cdot(1+\varepsilon)^\dagger} = \sqrt{\frac{1}{2}(1+1)} = 1$$

and

$$||Y|| = \left|\left|\frac{1}{\sqrt{2}}(1-\varepsilon)\right|\right| = \sqrt{\frac{1}{2}(1-\varepsilon)\cdot(1-\varepsilon)^\dagger} = \sqrt{\frac{1}{2}(1+1)} = 1.$$

These give $||X||\,||Y|| = 1$. Whereas for the left-hand side we have

$$||XY|| = \left|\left|\frac{1}{2}(1+\varepsilon)(1-\varepsilon)\right|\right| = \frac{1}{2}\left|\left|(1-\varepsilon^2)\right|\right| = ||(1-1)|| = 0,$$

where $\varepsilon^2 = 1$ is used. Thus we obtain $0 = ||XY|| \neq ||X||\,||Y|| = 1$.

Removing Inconsistency from the Counterexample

$$||X|| = \left|\left|\frac{1}{\sqrt{2}}(1+\varepsilon)\right|\right| = \sqrt{\frac{1}{2}(1+\varepsilon)(1+\varepsilon)^{\dagger}} = \sqrt{1+\varepsilon} \neq 0\,,$$

$$||Y|| = \left|\left|\frac{1}{\sqrt{2}}(1-\varepsilon)\right|\right| = \sqrt{\frac{1}{2}(1-\varepsilon)(1-\varepsilon)^{\dagger}} = \sqrt{1-\varepsilon} \neq 0,$$

where $\varepsilon^{\dagger} = \varepsilon$, $\varepsilon^2 = 1$, and geometric products instead of scalar products are used, reducing the right-hand side of norm relation to

$$||X||\,||Y|| = \left(\sqrt{1+\varepsilon}\right)\left(\sqrt{1-\varepsilon}\right) = \sqrt{(1-\varepsilon)(1+\varepsilon)} = \sqrt{1-\varepsilon^2} = 0.$$

But, as we noted, the left-hand side of norm relation is also zero:

$$||XY|| = \left|\left|\frac{1}{2}(1+\varepsilon)(1-\varepsilon)\right|\right| = \frac{1}{2}\,||(1-\varepsilon^2)|| = \frac{1}{2}||(1-1)|| = 0.$$

Thus, the norm relation $||XY|| = |||X||\,||Y||$ holds $\forall\ X, Y \in \mathcal{K}^{\lambda}$.

Algebraic Representation Space of S^3 is S^7

$$\mathcal{K}^\lambda \supset \widetilde{S^7} := \left\{ \mathbb{Q}_z := \mathbf{q}_r + \mathbf{q}_d\,\varepsilon \;\middle|\; ||\mathbb{Q}_z|| = \sqrt{\mathbb{Q}_z\mathbb{Q}_z^\dagger} = \sqrt{\varrho_c + \sigma_c\,\varepsilon} \right\},$$

where $\mathbb{Q}_z\mathbb{Q}_z^\dagger = \varrho_c + \sigma_c\,\varepsilon \quad \longleftarrow$ resembles a split complex number

$$= \left(\mathbf{q}_r\,\mathbf{q}_r^\dagger + \mathbf{q}_d\,\mathbf{q}_d^\dagger\right) + \left(\mathbf{q}_r\,\mathbf{q}_d^\dagger + \mathbf{q}_d\,\mathbf{q}_r^\dagger\right)\varepsilon$$

$$= \left(\varrho_r^2 + \varrho_d^2\right) + \left(-2\,q_0q_7 + 2\,\lambda\,q_1q_6 + 2\,\lambda\,q_2q_5 + 2\,\lambda\,q_3q_4\right)\varepsilon.$$

Setting $\boxed{\mathbf{q}_r\,\mathbf{q}_d^\dagger + \mathbf{q}_d\,\mathbf{q}_r^\dagger = 0}$ as **a special case**, the product $\mathbb{Q}_z\mathbb{Q}_z^\dagger$ reduces to a scalar quantity, and the quantity from norm relation,

$$\left\{\left(\varrho_{r1}^2 + \varrho_{d1}^2\right)\left(\varrho_{r2}^2 + \varrho_{d2}^2\right) + \left(\mathbf{q}_{r1}\,\mathbf{q}_{d1}^\dagger + \mathbf{q}_{d1}\,\mathbf{q}_{r1}^\dagger\right)\left(\mathbf{q}_{r2}\,\mathbf{q}_{d2}^\dagger + \mathbf{q}_{d2}\,\mathbf{q}_{r2}^\dagger\right)\right\}$$
$$+\left\{\left(\varrho_{r1}^2 + \varrho_{d1}^2\right)\left(\mathbf{q}_{r2}\,\mathbf{q}_{d2}^\dagger + \mathbf{q}_{d2}\,\mathbf{q}_{r2}^\dagger\right) + \left(\varrho_{r2}^2 + \varrho_{d2}^2\right)\left(\mathbf{q}_{r1}\,\mathbf{q}_{d1}^\dagger + \mathbf{q}_{d1}\,\mathbf{q}_{r1}^\dagger\right)\right\}\varepsilon,$$

also reduces to a scalar, which thus reduces the norm relation to

$$||\mathbb{Q}_{z1}\,\mathbb{Q}_{z2}|| = \sqrt{\left(\varrho_{r1}^2 + \varrho_{d1}^2\right)\left(\varrho_{r2}^2 + \varrho_{d2}^2\right)} = ||\mathbb{Q}_{z1}||\;||\mathbb{Q}_{z2}||,$$ giving

$$\boxed{\mathcal{K}^\lambda \supset S^7 := \left\{ \mathbb{Q}_z := \mathbf{q}_r + \mathbf{q}_d\,\varepsilon \;\middle|\; ||\mathbb{Q}_z|| = \sqrt{\mathbb{Q}_z\cdot\mathbb{Q}_z^\dagger} = \sqrt{\varrho_r^2 + \varrho_d^2} \right\}.}$$

The General Theorem — Correlations within S^7

The quantum mechanical correlations predicted by any arbitrary quantum state can be deterministically understood as classical, local, and realistic correlations among the limiting scalar points of values ± 1 of the 7-sphere constructed above, if these points are specified by manifestly local-realistic functions of the form

$$S^7 \ni \mathscr{A}(\mathbf{a},\, \lambda^k) := \lim_{\substack{\mathbf{s}_{r1} \to \mathbf{a}_r \\ \mathbf{s}_{d1} \to \mathbf{a}_d}} \left\{ \pm\, \mathbf{D}(\mathbf{a}_r,\, \mathbf{a}_d,\, 0)\, \mathbf{N}(\mathbf{s}_{r1},\, \mathbf{s}_{d1},\, 0,\, \lambda^k) \right\}$$

$$\xrightarrow[\substack{\mathbf{s}_{r1} \to \mathbf{a}_r \\ \mathbf{s}_{d1} \to \mathbf{a}_d}]{} \begin{cases} \mp 1 & \text{if} \quad \lambda^k = +1 \\ \pm 1 & \text{if} \quad \lambda^k = -1 \end{cases}, \text{ with } \left\langle \mathscr{A}(\mathbf{a},\, \lambda^k) \right\rangle = 0$$

and

$$S^7 \ni \mathscr{B}(\mathbf{b},\, \lambda^k) := \lim_{\substack{\mathbf{s}_{r2} \to \mathbf{b}_r \\ \mathbf{s}_{d2} \to \mathbf{b}_d}} \left\{ \mp\, \mathbf{D}(\mathbf{b}_r,\, \mathbf{b}_d,\, 0)\, \mathbf{N}(\mathbf{s}_{r2},\, \mathbf{s}_{d2},\, 0,\, \lambda^k) \right\}$$

$$\xrightarrow[\substack{\mathbf{s}_{r2} \to \mathbf{b}_r \\ \mathbf{s}_{d2} \to \mathbf{b}_d}]{} \begin{cases} \pm 1 & \text{if} \quad \lambda^k = +1 \\ \mp 1 & \text{if} \quad \lambda^k = -1 \end{cases}, \text{ with } \left\langle \mathscr{B}(\mathbf{b},\, \lambda^k) \right\rangle = 0,$$

where the orientation $\lambda = \pm 1$ of S^7 is assumed to be a fair coin. The proof of this theorem is given in DOI: 10.1098/rsos.180526.

Two Special Cases are Explicitly Computed in *RSOS*

For the special case of the two-particle entangled singlet state

$$|\Psi_{\mathbf{z}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{z},\,+\rangle_1 \otimes |\mathbf{z},\,-\rangle_2 \,-\, |\mathbf{z},\,-\rangle_1 \otimes |\mathbf{z},\,+\rangle_2\Big\}$$

the strong sinusoidal correlations are reproduced within S^7 exactly:

$$\mathcal{E}^{\rm Bell}_{L.R.}(\mathbf{a},\,\mathbf{b}) \,=\, \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\,\mathscr{A}(\mathbf{a}\,,\,\lambda^k)\;\mathscr{B}(\mathbf{b}\,,\,\lambda^k)\right] = -\,\mathbf{a}\cdot\mathbf{b}\,.$$

And for the four-particle GHZ (Greenberger-Horne-Zeilinger) state

$$|\Psi_{\mathbf{z}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{z},+\rangle_1\otimes|\mathbf{z},+\rangle_2 \otimes |\mathbf{z},-\rangle_3 \otimes |\mathbf{z},-\rangle_4 \\ -\,|\mathbf{z},-\rangle_1 \otimes |\mathbf{z},-\rangle_2 \otimes |\mathbf{z},+\rangle_3 \otimes |\mathbf{z},+\rangle_4\Big\}$$

the quantum mechanical correlations are again reproduced exactly:

$$\mathcal{E}^{\rm GHZ}_{L.R.}(\mathbf{a},\,\mathbf{b},\,\mathbf{c},\,\mathbf{d}) = \,\cos\theta_{\mathbf{a}}\,\cos\theta_{\mathbf{b}}\,\cos\theta_{\mathbf{c}}\,\cos\theta_{\mathbf{d}} \\ -\,\sin\theta_{\mathbf{a}}\,\sin\theta_{\mathbf{b}}\,\sin\theta_{\mathbf{c}}\,\sin\theta_{\mathbf{d}}\,\cos\left(\,\phi_{\mathbf{a}}\,+\,\phi_{\mathbf{b}}\,-\,\phi_{\mathbf{c}}\,-\,\phi_{\mathbf{d}}\,\right).$$

The S^7 constructed above captures the spinorial properties of the 3D physical space precisely, reproducing *all* quantum correlations.

Quantum Correlations are Weaved by the Spinors of the Euclidean Primitives

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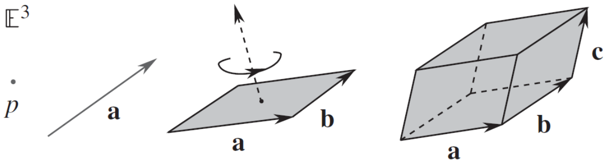

AGACSE 2021 – Conference in Honor of Prof. David Hestenes

07 September 2021 – Brno, Czech Republic

Refs: DOI: 10.1098/rsos.180526, *Royal Society Open Science*
DOI: 10.1109/ACCESS.2019.2941275, *IEEE Access*
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