# Geometric Solution of the Hierarchy Problem by Means of Einstein-Cartan Torsion

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Two of the major open questions in particle physics are: (1) Why are there no elementary fermionic particles observed in the mass-energy range between the electroweak scale and the Planck scale? And (2), what mechanical energy may be counterbalancing the divergent electrostatic and strong force energies of point-like charged fermions in the vicinity of the Planck scale? In this paper, using a hitherto unrecognized mechanism derived from the non-linear amelioration of Dirac equation known as the Hehl-Datta equation within Einstein-Cartan-Sciama-Kibble extension of general relativity, we present detailed numerical estimates suggesting that the mechanical energy arising from the gravity-induced four-fermion self-interaction in this theory can address both of these questions in tandem.

#### I. INTRODUCTION

For over a century Einstein's theory of gravity has provided remarkably accurate and precise predictions for the behaviour of macroscopic bodies within our cosmos. For the elementary particles in the quantum realm, however, Einstein-Cartan theory of gravity may be more appropriate, because it incorporates spinors and associated torsion within a covariant description [1][2]. For this reason there has been considerable interest in Einstein-Cartan theory, in the light of the field equations proposed by Sciama [3] and Kibble [4]. For example, in a series of papers Poplawski has argued that Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity [5] solves many longstanding problems in physics [6][7][8][9]. His concern has been to avoid singularities endemic in general relativity by proposing that our observed universe is perhaps a black hole within a larger universe [7]. Our concern, on the other hand, is to point out using numerical estimates that ECSK theory also offers solutions to two longstanding problems in particle physics.

The first of these problems can be traced back to the fact that gravity is a considerably weaker "force" compared to the other forces. When Newton's gravitational constant is combined with the speed of light and Planck's constant, one arrives at the energy scale of  $\sim 10^{19}$  GeV, which is some 17 orders of magnitude larger than the heaviest known elementary fermion (the top quark) observed at the mass-energy of  $\sim 175$  GeV. Thus there is a difference of some 17 orders of magnitude between the electroweak scale and the Planck scale. There have been many attempts to explain this difference, but none is as simple as our explanation based on the torsion contributions within the ECSK theory.

The second problem we address here concerns the well known fact that as we approach the Planck length,  $\sim 10^{-35}$  m, the electrostatic and strong force self-energies of point-like fermions become divergent. We will show, however, that torsion contributions within the ECSK theory resolves this difficulty as well, at least numerically, by counterbalancing the divergent electrostatic and strong force energy densities near the Planck scale. In fact, the negative torsion energy associated with the spin angular momentum of elementary fermions may well be the long sought after mechanical energy that counteracts the divergent positive energies stemming from their electrostatic and strong nuclear charges.

## II. STATIC COUNTERPART OF THE HEHL-DATTA EQUATION

The ECSK theory of gravity is an extension of general relativity allowing spacetime to have torsion in addition to curvature, where torsion is determined by the density of intrinsic angular momentum, reminiscent of the quantummechanical spin [1][2][3][4][5][6][7][8][9][10][11][12][13][14][15][16]. As in general relativity, the gravitational Lagrangian density in the ECSK theory is proportional to the curvature scalar. But unlike in general relativity, the affine connection  $\Gamma_{ij}^{\ k}$  is not restricted to be symmetric. Instead, the antisymmetric part of the connection,  $S_{ij}^{\ k} = \Gamma_{[ij]}^{\ k}$  (*i.e.*,

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the torsion tensor), is regarded as a dynamical variable similar to the metric tensor  $g_{ij}$  in general relativity. Then, variation of the total action for the gravitational field and matter with respect to the metric tensor gives Einstein-type field equations that relate the curvature to the dynamical energy-momentum tensor  $T_{ij} = (2/\sqrt{-g})\delta \mathfrak{L}/\delta g^{ij}$ , where  $\mathfrak{L}$  is the matter Lagrangian density. On the other hand, variation of the total action with respect to the torsion tensor gives the Cartan equations for the spin tensor of matter [5]:

$$s^{ijk} = \frac{1}{\kappa} S^{[ijk]}, \quad \text{where } \kappa = \frac{8\pi G}{c^4}.$$
 (1)

Thus ECSK theory of gravity extends general relativity to include intrinsic spin of matter, with fermionic fields such as those of quarks and leptons providing natural sources of torsion. Torsion, in turn, modifies the Dirac equation for elementary fermions by adding to it a cubic term in the spinor fields, as observed by Kibble, Hehl and Datta [1][4][5].

It is this nonlinear Hehl-Datta equation that provides the theoretical background for our proposal. The cubic term in this equation corresponds to an axial-axial four-fermion self-interaction in the matter Lagrangian, which, among other things, generates a spinor-dependent vacuum-energy term in the energy-momentum tensor (see, for example, Ref. [13]). The torsion tensor  $S^{k}_{ij}$  appears in the matter Lagrangian via covariant derivative of a Dirac spinor with respect to the affine connection. The spin tensor for the Dirac spinor  $\psi$  then turns out to be totally antisymmetric:

$$s^{ijk} = -\frac{i\hbar c}{4}\bar{\psi}\gamma^{[i}\gamma^{j}\gamma^{k]}\psi, \qquad (2)$$

where  $\bar{\psi} \equiv \psi^{\dagger} \gamma^{0} := (\psi_{1}^{*}, \psi_{2}^{*}, -\psi_{3}^{*}, -\psi_{4}^{*})$  is the Dirac adjoint of  $\psi$  and  $\gamma^{i}$  are the Dirac matrices:  $\gamma^{(i} \gamma^{j)} = 2g^{ij}$ . The Cartan equations (1) render the torsion tensor to be quadratic in spinor fields. Substituting it into the Dirac equation in the Riemann-Cartan spacetime with metric signature (+, -, -, -) gives the cubic Hehl-Datta equation [1][4][5]:

$$-i\hbar\gamma^{k}\psi_{k} = mc\psi + \frac{3\kappa\hbar^{2}c}{8}\left(\bar{\psi}\gamma^{5}\gamma_{k}\psi\right)\gamma^{5}\gamma^{k}\psi, \qquad (3)$$

where the colon denotes a general-relativistic covariant derivative with respect to the Christoffel symbols, and m is the mass of the spinor. The Hehl-Datta equation (3) and its adjoint can be obtained by varying the following action with respect to  $\bar{\psi}$  and  $\psi$  (respectively), without varying it with respect to the metric tensor or the torsion tensor [13]:

$$\mathcal{S} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{\kappa} R - \frac{i\hbar c}{2} (\bar{\psi}\gamma^k \psi_{:k} - \bar{\psi}_{:k}\gamma^k \psi) - mc^2 \bar{\psi}\psi - \frac{3\kappa\hbar^2 c^2}{16} (\bar{\psi}\gamma^5 \gamma_k \psi) (\bar{\psi}\gamma^5 \gamma^k \psi) \right\}.$$
(4)

The last term in this action corresponds to the effective axial-axial, four-fermion self-interaction mentioned above:

$$\mathfrak{L}_{AA} = -\sqrt{-g} \,\frac{3\kappa\hbar^2 c^2}{16} (\bar{\psi}\gamma^5\gamma_k\psi)(\bar{\psi}\gamma^5\gamma^k\psi). \tag{5}$$

This self-interaction term is not renormalizable. But it is an *effective* Lagrangian density in which only the metric and spinor fields are dynamical variables. The original Lagrangian density for a Dirac field in which the torsion tensor is also a dynamical variable (giving the Hehl-Datta equation), *is* renormalizable, since it is quadratic in spinor fields. But as we will see renormalization may not be required, if ECSK gravity turns out to be what is realized in Nature.

Before proceeding further we note that the above action is not the most general possible action within the present context. In addition to the axial-axial term, an axial-vector and a vector-vector terms can be added to the action, albeit as non-minimal couplings (see, for example, Ref. [15]). However, it has been argued in Ref. [13] that minimal coupling is the most natural coupling of fermions to gravity because non-minimal couplings are sourced by components of the torsion that do not appear naturally in the models of spinning matter. For this reason we will confine our treatment to the minimal coupling of fermions to gravity and the corresponding Hehl-Datta equation, while recognizing that strictly speaking our neglect of non-minimal couplings amounts to an approximation, albeit a rather good one.

Moving forward to our goal of numerical estimates, if we require the action (4) to be invariant under local U(1) phase transformations, then  $\psi_{k}$  transforms to  $\psi_{k} + iqA_k\psi/\hbar$  for a charge q and a gauge field  $A_k$ , and eq. (3) generalizes to

$$-i\hbar\gamma^{k}\psi_{k} + q\gamma^{k}A_{k}\psi = mc\psi + \frac{3\kappa\hbar^{2}c}{8}\left(\bar{\psi}\gamma^{5}\gamma_{k}\psi\right)\gamma^{5}\gamma^{k}\psi.$$
(6)

In the rest frame of the particle and anti-particle, with the metric signature (+, -, -, -), this equation simplifies to

$$-i\hbar\gamma^{0}\frac{\partial\psi}{\partial t} + qcA_{0}\gamma^{0}\psi = mc^{2}\psi + \frac{3\kappa\hbar^{2}c^{2}}{8}\left(\bar{\psi}\gamma^{5}\gamma_{0}\psi\right)\gamma^{5}\gamma^{0}\psi, \qquad (7)$$

which can be further simplified to

$$-i\hbar \begin{pmatrix} +\frac{\partial\psi_1}{\partial t} \\ +\frac{\partial\psi_2}{\partial t} \\ -\frac{\partial\psi_3}{\partial t} \\ -\frac{\partial\psi_4}{\partial t} \end{pmatrix} + qcA_0 \begin{pmatrix} +\psi_1 \\ +\psi_2 \\ -\psi_3 \\ -\psi_4 \end{pmatrix} = mc^2 \begin{pmatrix} +\psi_1 \\ +\psi_2 \\ +\psi_3 \\ +\psi_4 \end{pmatrix} - \frac{3\kappa\hbar^2c^2}{8} \left\{ \psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^* \right\} \begin{pmatrix} -\psi_3 \\ -\psi_4 \\ +\psi_1 \\ +\psi_2 \end{pmatrix}, \quad (8)$$

where we have used

$$\gamma^{0} = \begin{pmatrix} +1 & 0 & 0 & 0\\ 0 & +1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad \gamma^{5} = \begin{pmatrix} 0 & 0 & +1 & 0\\ 0 & 0 & 0 & +1\\ +1 & 0 & 0 & 0\\ 0 & +1 & 0 & 0 \end{pmatrix}.$$
(9)

If we now represent the particles and anti-particles with two-component spinors  $\psi_a$  and  $\psi_b$ , respectively [17], where

$$\psi_a := \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \text{and} \quad \psi_b := \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}$$
(10)

are the two-component spinors constituting the four-component Dirac spinor, then the above equation can be written as two *coupled* partial differential equations:

$$-i\hbar \frac{\partial \psi_a}{\partial t} + qcA_0 \,\psi_a = mc^2 \,\psi_a + \frac{3\kappa\hbar^2 c^2}{8} \left\{ \psi_1^* \psi_3 + \psi_2^* \psi_4 + \psi_1 \psi_3^* + \psi_2 \psi_4^* \right\} \,\psi_b \tag{11}$$

$$+i\hbar \frac{\partial \psi_b}{\partial t} - qcA_0 \psi_b = mc^2 \psi_b - \frac{3\kappa\hbar^2 c^2}{8} \left\{ \psi_1^* \psi_3 + \psi_2^* \psi_4 + \psi_1 \psi_3^* + \psi_2 \psi_4^* \right\} \psi_a \,. \tag{12}$$

Unlike the case in Dirac equation, these equations for the spinors  $\psi_a$  and  $\psi_b$  are coupled equations even in the rest frame. They decouple in the limit when the torsion-induced axial-axial four-fermion interaction is negligible. On the other hand, at low energies it is reasonable to assume that, in analogy with the Dirac spinors in flat spacetime, the above two-component spinors for free particles decouple in the rest frame, admitting plane wave solutions of the form

$$\psi_a(t) = e^{-i(mc^2/\hbar)t}\psi_a(0) \quad \text{and} \quad \psi_b(t) = e^{+i(mc^2/\hbar)t}\psi_b(0),$$
(13)

together with

$$\psi_b(t) = e^{+i(2mc^2/\hbar)t} \,\psi_a(t),\tag{14}$$

so that initially they are equal to each other [17]:

$$\psi_b(0) = \psi_a(0). \tag{15}$$

Substitution of this form of solutions into eqs. (11) and (12) reduces these equations to the following two equations:

$$-mc^{2}\psi_{a}(t) + qcA_{0}\psi_{a}(t) = mc^{2}\psi_{a}(t) + \frac{3\kappa\hbar^{2}c^{2}}{4}\left\{\cos\left(\frac{2mc^{2}}{\hbar}t\right)|\psi_{a}(0)|^{2}\right\}\psi_{b}(t)$$
(16)

and 
$$-mc^2 \psi_b(t) - qcA_0 \psi_b(t) = mc^2 \psi_b(t) - \frac{3\kappa\hbar^2 c^2}{4} \left\{ \cos\left(\frac{2mc^2}{\hbar}t\right) |\psi_b(0)|^2 \right\} \psi_a(t) .$$
 (17)

Now we are only concerned about the static scenario, for which (*i.e.*, at time t = 0) these pair of equations reduce to

$$+\frac{qcA_0}{2}\psi_a(0) = mc^2\psi_a(0) + \frac{3\kappa\hbar^2c^2}{8}|\psi_a(0)|^2\psi_b(0)$$
(18)

and 
$$-\frac{qcA_0}{2}\psi_b(0) = mc^2\psi_b(0) - \frac{3\kappa\hbar^2c^2}{8}|\psi_b(0)|^2\psi_a(0).$$
 (19)

In order to obtain scalar counterparts of these (now essentially decoupled) equations, we multiply them through from the left by  $\bar{\psi}_a(0) = (\psi_1^*(0), \psi_2^*(0))$  and  $\bar{\psi}_b(0) = (\psi_3^*(0), \psi_4^*(0))$ , respectively, and – in the light of eq. (15) – arrive at

$$+\frac{qcA_0}{2}|\psi_a(0)|^2 - \frac{3\kappa\hbar^2c^2}{8}|\psi_a(0)|^4 = mc^2|\psi_a(0)|^2$$
(20)

and 
$$-\frac{qcA_0}{2}|\psi_b(0)|^2 + \frac{3\kappa\hbar^2c^2}{8}|\psi_b(0)|^4 = mc^2|\psi_b(0)|^2.$$
 (21)

Substituting in SI units for the scalar field  $A_0 = V/c = q/(c 4\pi\varepsilon_0 r)$  in the Lorentz gauge (where V is the electric potential), and for  $\kappa = \frac{8\pi G}{c^4}$ , together with the probabilities  $|\psi_b(0)|^2 = |\psi_a(0)|^2 \sim 1/r^3$  of finding the particles in the volume  $r^3$  [cf. eq. (15)], we finally arrive at our central equations, which hold at least for t = 0, for any electroweak fermion of charge q and mass m and its anti-particle in the Riemann-Cartan spacetime:

$$\frac{q^2}{8\pi\varepsilon_0 r^4} - \frac{3\pi G\hbar^2}{c^2 r^6} = \frac{mc^2}{r^3}$$
(22)

and 
$$\frac{3\pi G\hbar^2}{c^2 r^6} - \frac{q^2}{8\pi\varepsilon_0 r^4} = \frac{mc^2}{r^3},$$
 (23)

where r is the radial distance from q and the two equations correspond to the particle and anti-particle, respectively. Note that, as one would expect, if we multiply through each of these two equations by  $r^3$ , then for fermion anti-fermion pair annihilation the terms on the LHS of the equations cancel out, leaving  $2mc^2$  in energy to fly off as photons.

It is also worth noting that without ameliorating the Dirac equation with a cubic term, eq. (22) would reduce for an electron to  $\alpha \hbar/2 r_e = m_e c$ , giving  $r_e = (\alpha \hbar/2 m_e c) \sim 10^{-15}$  m, where  $\alpha = e^2/(4\pi \varepsilon_0 \hbar c)$  is the fine structure constant. This is one half of the classical electron radius. Experimental evidence, however, suggests that electron radius is much smaller. As we shall see, our calculations with the cubic term included predicts the electron radius to be of the order of  $10^{-34}$  m, which is closer to the Planck length. This may turn out to be the correct value of the electron radius.

Needless to say, what we have presented above is a derivation of eq. (22) within a theory that may be viewed as a semi-classical theory of Dirac fields in a Riemann-Cartan spacetime [4][5]. It can be interpreted also as a theory of gravity-induced four-fermion self-interaction within standard general relativity [1][5]. A possible second-quantized generalization of this theory is beyond the scope of our paper. However, any such generalization must necessarily reproduce the Hehl-Datta equation (3) for single fermions even at reasonably high energies, just as Dirac equation remains valid for single fermions at high energies [16]. It is therefore not unreasonable to base our numerical estimates below on eq. (22) derived above. We shall soon see that this equation is both necessary and sufficient for our purposes.

Finally, it is important to note here that, despite the appearance of four spinors in the interaction term of eq. (4), it describes the self-interaction of a *single* fermion, of range  $\sim 10^{-31}$  m, not mutual interactions among the spins of four distinct fermions. That is to say, it does not describe a "spin field" of some sort as a carrier of a new interaction [5]. If, however, one insists on interpreting the interaction term in eq. (4) as describing interactions among four distinct fermions, then the mass of the corresponding exchange boson would have to exceed  $10^{14}$  GeV, which is evidently quite unreasonable. What is more, as we shall soon see, within our scheme any corrections due to vacuum polarization are automatically compensated for in the production of electroweak mass-energy, dictated by eqs. (22) and (23) above.

#### **III. PARTICLE MASSES VIA TORSION ENERGY CONTRIBUTION**

For our numerical analysis it is instructive to bring out the physical content of eq. (22) in purely classical terms. To this end, recall that when the self-energy of an electron is attributed solely to its electrostatic content, it is found that that energy is divergent, provided we assume the electron to be a point particle. This energy is called "self-energy", because it arises from the interaction of the charge of the electron with the electrostatic field that it itself is creating. With the help our result (22) above we can avoid this fundamental difficultly as follows. Multiplying through eq. (22) with  $3/4\pi$ , it can be written as

$$\left(\frac{q^2}{8\pi\varepsilon_0 r}\right)\frac{3}{4\pi r^3} - \left(\frac{3\pi G\hbar^2}{c^2 r^3}\right)\frac{3}{4\pi r^3} = (mc^2)\frac{3}{4\pi r^3}.$$
(24)

With  $4\pi r^3/3$  recognized as a volume of a sphere of radius r, it is now easy to recognize each quantity in the parenthesis in this equation as energy, and each term as the corresponding energy density. Now the first term on the left of the equation is three times the electrostatic energy density at a distance r from the charge, with the latter given by [18]

$$u_{\text{static}} = \frac{q^2}{32\pi^2\varepsilon_0 r^4} \,. \tag{25}$$

And the quantity in the first parenthesis is the total energy of a "thin" spherical shell of charge q at a distance r [18]:

$$U_{\rm sphere} = \frac{q^2}{8\pi\varepsilon_0 r} \,. \tag{26}$$

Unfortunately the first term in eq. (24) diverges as  $r \to 0$ . But if we cut-off r at the Planck length,  $l_P = \sqrt{G\hbar/c^3}$ , then by setting q = -e for an electron we obtain

$$\frac{3e^2}{32\pi^2\varepsilon_0 r^4} \approx 2.5190 \times 10^{120} \text{ GeV m}^{-3}.$$
(27)

Although finite, this is still an extremely large energy density. But such a large energy density for charged leptons is never realized in Nature. A natural question then is: Is there a negative mechanical energy density that cancels out most of this energy to produce the observed rest mass-energy of leptons? We believe the answer lies in the second term of eq. (24), which – as we saw above – arises from the non-linear amelioration of the Dirac equation within the ECSK theory. Indeed, if we again set the Planck length cut-off for r in the second term of eq. (24), then we obtain

$$-\frac{9\,G\hbar^2}{4\,c^2\,l_P^6} \approx -\,6.5067 \times 10^{123} \text{ GeV m}^{-3}.$$
(28)

Comparing this value with the electrostatic energy density at the Planck length cut-off estimated in eq. (27) we see at once that the torsion-induced mechanical energy (28) can indeed counterbalance the huge electrostatic energy. This is a surprising observation, considering the widespread belief that "the numerical differences which arise [between GR and ECSK theories] are normally very small, so that the advantages of including torsion are entirely theoretical" [16].

Moving forward to our goal of numerical estimates, let us note that whenever terms quadratic in spin happen to be negligible, then the ECSK theory is observationally indistinguishable from general relativity. Therefore, for postgeneral-relativistic effects, the density of spin-squared has to be comparable to the density of mass. The corresponding characteristic length scale, say for a nucleon, is referred to as the Cartan or Einstein-Cartan radius, defined as [2][16]

$$r_{Cart} \approx (l_P^2 \,\lambda_C)^{\frac{1}{3}},\tag{29}$$

where  $\lambda_C$  is the Compton wavelength of the nucleon. Now it has been noted by Poplawski [6][7][8][9] that quantum field theory based on the Hehl-Datta equation may avert divergent integrals normally encountered in calculating radiative corrections, by self-regulating propagators. Moreover, the multipole expansion applied to Dirac fields within the ECSK theory shows that such fields cannot form singular, point-like configurations because these configurations would violate the conservation law for the spin density, and thus the Bianchi identities. These fields in fact describe non-singular particles whose spatial dimensions are at least of the order of their Cartan radii, defined by the condition

$$\epsilon \sim \kappa s^2,$$
 (30)

where  $\sqrt{s^2} \sim \hbar c |\psi|^2$  is the spin density,  $\epsilon \sim mc^2 |\psi|^2$  is the rest energy density, and  $|\psi|^2 \sim 1/r^3$  is the probability density, giving the radius (29). Consequently, at the least the de Broglie energy associated with the Cartan radius of a fermion (which is approximately  $10^{-27}$  m for an electron) may introduce an effective ultraviolet cutoff for it in quantum field theory in the ECKS spacetime. The avoidance of divergences in radiative corrections in quantum fields may thus come from spacetime torsion originating from intrinsic spin. Poplawski and others, however, took  $\epsilon$  to be the mass-energy density of the fermion to arrive at the Cartan radius (29). But it is easy to work out from the first term of our eq. (22) that at the Cartan radius the electrostatic energy density for an electron is still extremely large:

$$\frac{3\,\alpha\hbar c}{8\pi\,(10^{-27}m)^4} \approx 1.7188 \times 10^{89} \text{ GeV m}^{-3}.$$
(31)

For this reason it is not correct to identify  $\epsilon$  with the rest mass-energy density, which is  $\approx 5.1099 \times 10^{77}$  GeV m<sup>-3</sup> for an electron at the Cartan radius. The electrostatic energy density of an electron is thus eleven orders of magnitude higher. Therefore  $\epsilon$  is better identified with the electrostatic energy density (31), provided most of it is canceled out.

If in eq. (22) we set the electrostatic energy density appearing in its first term to be equal to the spin energy density induced by the four-fermion self-interaction appearing in its second term and solve for r, then we obtain

$$r_t = \sqrt{\frac{6\pi}{\alpha}} l_P \approx 8.2143 \times 10^{-34} \,\mathrm{m}\,.$$
 (32)

Which is about fifty one times larger than the Planck length, and is a remarkably simple constant in terms of the Planck length and the fine structure constant. According to eq. (22), this is the effective radius at which energy density due to spin density should completely compensate the huge electrostatic energy seen in (27). In our view this is the correct Cartan radius, at least for the charged leptons, that may still provide a plausible mechanism for averting singularities, since it is closer to the Planck length. It is important to note, however, that these huge energy densities never actually occur in Nature, because according to our eq. (22) they are automatically compensated. The physical mechanism described above is simply to enable extraction of the radius  $r_t$  for different charged fermions.

In order to obtain an observed mass-energy for the elementary fermions, we now posit that there is a very small difference between the radii of their electrostatic energy density and their spin energy density:  $\Delta r := |r_x - r_t|$ . We do not propose a specific reason for this difference, but one possible reason may be our neglect to include axial-vector and vector-vector interactions in the action (4) for the derivation of Hehl-Datta equation, as we discussed earlier [13][15]. While exclusion of such non-minimal couplings may be justified on physical grounds, their inclusion in the derivation of eq. (22) would not have allowed us to decouple the equations of motion (16) and (17) for particles and anti-particles at t = 0. Consequently, in that case the probability densities such as  $|\psi_a(0)|^2 \sim 1/r^3$  could not be assumed to be exactly the same for the electrostatic term and the spin density term in those equations. Perhaps a second-quantized generalization of the ECSK-Dirac theory would eventually lead us to a better understanding of the origin(s) of  $\Delta r$ .

In order to approximate the difference  $\Delta r$ , we hold the radius for the spin energy density to that of the "cancellation radius"  $r_t$ , because this radius is constant for a given charged lepton, and because we expect spin energy density to be the same for all charged leptons. We then vary the radius for the electrostatic and the rest mass-energy densities, which we take to be the same. Using eq. (22), this leads us to the following formula for our numerical estimates:

$$\frac{\alpha\hbar c}{2\,r_x^4} - \frac{3\,\pi G\hbar^2}{c^2\,r_t^6} = \frac{m_x c^2}{r_x^3}\,.\tag{33}$$

As shown in the Appendix below, we were able to find solutions for  $r_x$  for the charged leptons using arbitrary precision in *Mathematica*. The first in our results listed below is the solution for  $r_t$  up to 20 significant figures. Then, using the same precision for comparison, we list the results for  $r_e$  for an electron,  $r_{\mu}$  for a muon, and  $r_{\tau}$  for a tauon, along with the anti-fermions using eq. (23):

$$\begin{split} r_t &= 8.2143011998060519095 \times 10^{-34} \text{ m} \longrightarrow 0.0 \text{ MeV}, \\ r_{e-} &= 8.2143011998060519083 \times 10^{-34} \text{ m} \longrightarrow 0.511 \text{ MeV}, \\ r_{\mu-} &= 8.2143011998060518270 \times 10^{-34} \text{ m} \longrightarrow 106 \text{ MeV}, \\ r_{\tau-} &= 8.2143011998060505218 \times 10^{-34} \text{ m} \longrightarrow 1777 \text{ MeV}, \\ r_{e+} &= 8.2143011998060519107 \times 10^{-34} \text{ m} \longrightarrow 0.511 \text{ MeV}, \\ r_{\mu+} &= 8.2143011998060519920 \times 10^{-34} \text{ m} \longrightarrow 106 \text{ MeV}, \\ r_{\tau+} &= 8.2143011998060532972 \times 10^{-34} \text{ m} \longrightarrow 1777 \text{ MeV}. \end{split}$$

Evidently, very minute changes in the radii are seen to cause large changes in the observed rest mass-energies of the fermions. But as the differences in the radii go larger, the resultant mass-energies go higher, as one would expect. Needless to say, for the anti-leptons the results for  $r_x$  will be larger than  $r_t$  rather than smaller, by the same amount.

It seems extraordinary that Nature would subscribe to such tiny differences resulting from large number of significant figures, but that might explain why the underlying relationship between the observed values of the masses of the

elementary particles has remained elusive so far. In addition to the possible reasons for this mentioned above, it is not inconceivable that the difference between the spin energy density and the electrostatic energy density radii arises due to purely geometrical factors. We also suspect that there may possibly be some kind of symmetry breaking mechanism at work similar to the Higgs mechanism, and this symmetry breaking results in the observed mass-energy generation.

As a consistency check, let us verify that the tiny length differences seen above vanish,  $\Delta r \to 0$ , as the corresponding rest mass-energy differences tend to zero:  $\Delta E \to 0$ . To this end, we recast eq. (33) for arbitrary  $r_x$  in a form involving only rest mass-energy on the RHS as:

$$\frac{\alpha\hbar c}{2\,r_x} - \frac{3\pi G\hbar^2}{c^2\,r_t^6}\,r_x^3 = m_x c^2. \tag{34}$$

If we now set

$$A \equiv \frac{\alpha \hbar c}{2} \quad \text{and} \quad B \equiv \frac{3\pi G \hbar^2}{c^2} \,,$$
(35)

then, with  $\Delta E = m_x c^2$  and setting  $r_x = r_t$  as the cancellation radius for which  $\Delta E = 0$ , we obtain

$$r_t = \sqrt{B/A} \,. \tag{36}$$

This allows us to derive a general expression for  $r_x$  when  $\Delta E \neq 0$ :

$$\frac{A}{r_x} - \frac{A^3}{B^2} r_x^3 = \Delta E.$$
(37)

From this expression it is now easy to see that

$$\lim_{\Delta E \to 0} \left\{ \frac{A}{r_x} - \frac{A^3}{B^2} r_x^3 = \Delta E \right\} \implies r_x = \sqrt{B/A} = r_t , \qquad (38)$$

and conversely, using (36),

$$\lim_{r_x \to r_t} \left\{ \frac{A}{r_x} - \frac{B}{r_t^6} r_x^3 = \Delta E \right\} \implies \Delta E = 0.$$
(39)

Consequently, with  $\Delta r \equiv |r_t - r_x|$ , we see from the above limits that  $\Delta r \to 0$  as  $\Delta E \to 0$ , and vice versa.

As a rough estimate the calculation for the radius  $r_q$  of elementary quarks can be performed in a similar manner as that for electrons, since at such short distances the strong force reduces to a Coulomb-like force. One must also factor-in the electrostatic energy, so that a relationship like the following must be calculated, say, for the top quark:

$$\frac{2\,\alpha\hbar c}{9\,r_{qx}^4} + \frac{4\,\alpha_s\hbar c}{r_{qx}^4} - \frac{3\pi G}{c^4} \left(\frac{\hbar c}{r_{tq}^3}\right)^2 = \frac{m_t c^2}{r_{qx}^3}\,.$$
(40)

Here  $\alpha_s$  is the appropriate strong force coupling (we use 0.1), and we have used a generic expression for the total energy density of the strong field as a spherical shell less spherical symmetry to match the other terms. Needless to say, a cancellation radius different from that of the charged leptons has to be calculated first, by setting

$$\frac{2\alpha\hbar c}{9r_{tq}^4} + \frac{4\alpha_s\hbar c}{r_{tq}^4} = \frac{3\pi G}{c^4} \left(\frac{\hbar c}{r_{tq}^3}\right)^2. \tag{41}$$

A calculation of the radius for the top quark based on eq. (40) can be found in the Appendix. We expect it to be only a very rough estimate of the actual value of the radius. Since only one spin density is involved, the above calculation might be able to approximate the behaviour of the quarks. But there may be slightly different radial differences for the electrostatic part and the strong force part, which would make solving for those differences very difficult. The calculation of the radii  $r_q$  for the up and down quark will probably be problematic as well, since their masses are not well known. But if an underlying relationship is discovered, then that may help to know those masses better.

In a similar rough manner we approximate the radius  $r_n$  for neutrinos by replacing  $\alpha$  with the coupling for the weak force,  $\alpha_w \sim 1/29.5$ , in eq. (33) and using the mass-energy upper limit from Ref. [20]. The results are consistent with radius of  $\sim 10^{-34}$  m. It appears that our approximations are of the same order for all leptons and quarks, and that according to our rough estimates the size of all elementary fermions could be very close to the Planck length.

### IV. POSSIBLE SOLUTION OF THE HIERARCHY PROBLEM

As alluded to in the introduction, the Hierarchy Problem refers to the fact that gravitational interaction is extremely weak compared to the other known interactions in Nature. One way to appreciate this difference is by combining the Newton's gravitational constant G with the reduced Planck's constant  $\hbar$  and the speed of light c. The resulting mass scale is the Planck mass,  $m_P$ , which some have speculated to be associated with the existence of smallest possible black holes [7]. If we compare the Plank mass with the mass of the top quark (the heaviest known elementary particle),

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.1765 \times 10^{-8} \text{ kg},$$
$$m_t = \frac{173.21 \text{ GeV}}{c^2} \approx 3.1197 \times 10^{-25} \text{ kg},$$

then we see that there is some 17 orders of magnitude difference between them. This illustrates the enormous difference between the Planck scale and the electroweak scale. Many solutions have been proposed to explain this difference, such as supersymmetry and large extra dimensions, but none has been universally accepted, for one reason or another. Furthermore, recent experiments performed with the Large Hadron Collider are gradually ruling out some of these proposals. But regardless of the nature of any specific proposal, it is clear from the above values that predictions of numbers with at least 17 significant figures are necessary to successfully explain the difference between  $m_P$  and  $m_t$ .

We saw from our numerical demonstration in the previous section that within the ECSK theory minute changes in length can induce sizable changes in the observed masses of elementary particles, and that we do have numbers at our disposal with more than 17 significant figures for producing those masses. Moreover, all length changes occurring in our demonstration are taking place close to the Planck length. Thus, since we are "canceling out" near the Planck length to obtain masses down to the electroweak scale, ours is clearly a possible mechanism for resolving the Hierarchy Problem. Within the ECSK theory, which extends general relativity to include spin-induced torsion, gravitational effects near micro scales are not necessarily weak. On the other hand, since torsion is produced in the ECSK theory by the spin density of matter, it is mostly confined to that matter, and thus is a very short range effect, unlike the infinite range effect of Einstein's gravity produced by mass-energy. In fact the torsion field falls off as  $1/r^6$ , as shown in the calculations of Sect. III, since it is produced by spin density squared, confined to the matter distribution [9].

To compare the strengths of gravitational and torsion effects at various scales, we may define a mass-dependent dimensionless gravitational coupling constant,  $\frac{Gm^2}{\hbar c}$ , and evaluate it for the electron, top quark, and Planck masses:

$$\alpha_{G_e} = \frac{Gm_e^2}{\hbar c} \approx 1.7517 \times 10^{-45},$$
  

$$\alpha_{G_t} = \frac{Gm_t^2}{\hbar c} \approx 1.1620 \times 10^{-36},$$
  

$$\alpha_{G_P} = \frac{Gm_P^2}{\hbar c} = 1,$$
  

$$\alpha_e = \frac{e^2}{4\pi\varepsilon_0\hbar c} \simeq 7.2973 \times 10^{-3}.$$

Here  $\alpha_e$  is the electromagnetic coupling constant, or the fine structure constant. From these values we see that near the Planck scale the gravitational coupling is very strong compared to the electromagnetic coupling. However, as we noted above and in Sect. III, near the Planck scale torsion effects due to spin density are also very strong, albeit with opposite polarity compared to that of Einstein's gravity, akin to a kind of "anti-gravity" effect of a very short range.

For our demonstration above we have used electrostatic energy density and spin density for matter in a static approximation, for which the field equation within the ECSK theory reduces to  $G^{00} = T^{00}$ . A numerical estimate for  $G^{00}$  from the contributions of the electrostatic energy and spin density parts of  $T^{00}$  at our cancellation radius gives

$$G_{stat}^{00} = \frac{8\pi G}{c^4} \frac{\alpha \hbar c}{2 r_t^4} \approx +5.2614 \times 10^{61} \ m^{-2} \tag{42}$$

and 
$$G_{spin}^{00} = -\frac{8\pi G}{c^4} \frac{3\pi G\hbar^2}{c^2 r_t^6} \approx -5.2614 \times 10^{61} m^{-2}.$$
 (43)

Evidently, these field strengths at the cancellation radius are quite large even for a single electron. Fortunately they are never realized in Nature, because, as we can see, they cancel each other out to produce  $G_{net}^{00} = 0$ . On the other hand, if we use only the mass-energy density for electron at the cancellation radius, then we obtain  $G_{mass}^{00} \approx 3.0674 \times 10^{43} \text{ m}^{-2}$ , which is again some 18 orders of magnitude off the mark. What is more, the latter field strength does not fall off as fast as that due to the spin-induced torsion field. Thus it is reasonable to conclude that without the cancellation of divergent energies due to the four-fermion interaction we have explored here, our universe would be highly improbable.

# V. CONCLUDING REMARKS

In this paper we have addressed two longstanding questions in particle physics: (1) Why are there no elementary fermionic particles observed in the mass-energy range between the electro-weak scale and the Planck scale? And (2), what mechanical energy may be counterbalancing the divergent electrostatic and strong force energies of point-like charged fermions in the vicinity of the Planck scale? Using a hitherto unrecognized mechanism extracted from the well known Hehl-Datta equation, we have presented numerical estimates suggesting that the torsion contributions within the Einstein-Cartan-Sciama-Kibble extension of general relativity can address both of these questions in conjunction.

The first of these problems, the Hierarchy Problem, can be traced back to the extreme weakness of gravity compared to the other forces, inducing a difference of some 17 orders of magnitude between the electroweak scale and the Planck scale. There have been many attempts to explain this huge difference, but none is simpler than our explanation based on the spin induced torsion contributions within the ECSK theory of gravity. The second problem we addressed here concerns the well known divergences of the electrostatic and strong force self-energies of point-like fermions at short distances. We have demonstrated above, numerically, that torsion contributions within the ECSK theory resolves this difficulty as well, by counterbalancing the divergent electrostatic and strong force energies close to the Planck scale.

It is widely accepted that in the standard model of particle physics charged elementary fermions acquire masses via the Higgs mechanism. Within this mechanism, however, there is no satisfactory explanation for how the different couplings required for the fermions are produced to give the correct values of their masses. While the Higgs mechanism does bestow masses correctly to the heavy gauge bosons and a massless photon, and while our demonstration above does not furnish a fundamental explanation for the fermion masses either, we believe that what we have proposed in this paper is worthy of further research, since our proposal also offers a possible resolution of the Hierarchy Problem.

Needless to say, the geometrical cancellation mechanism for divergent energies we have proposed here also dispels the need for mass-renormalization, with our cancellation radius  $r_t$  acting as a natural cutoff radius taming the infinities. Thus both classical and quantum electrodynamics appears to be more complete with torsion contributions included.

#### Appendix: Calculations of Cancellation Radii using Wolfram Mathematica

In this appendix we explain how we used the arbitrary-precision in *Mathematica* to solve the numerical equations out to 22 significant figures. Each equation displayed below — derived from our central equation (22) — is simplified so that only the numerical factors have to be used, since the dimensional units cancel out, leaving lengths in meters. For decimal factors, the numbers must be padded out to 22 digits with zeros. Then the numerical part of electrostatic energy density is defined as A and the numerical part of spin energy density is defined as B, just as in eq. (35) above. These are then used throughout to perform the calculations. For the values of various physical constants involved in the calculations we have used the 2014 CODATA values, Ref. [19], and values from the Particle Data Group, Ref. [20].

### Calculation of the Cancellation Radius for Charged Leptons using Formula (22):

$$\frac{\alpha\hbar c}{2}r_t^2 - \frac{3\pi G\hbar^2}{c^2} = 0 \tag{A.1}$$

$$\begin{split} & ((2.9979245800000000000 \times 10^8)^2), 22]; \\ & N[\text{Solve}[A*r_t^2 - B == 0, r_t], 22] \, //\text{Last} \\ & \{r_t \to 8.2143011998060519095 \, \times \, 10^{-34}\} \end{split}$$

### Calculation of Radius $r_e$ of Electron and Positron

$$\frac{\alpha\hbar c}{2r_{e-}^4} - \frac{3\pi G\hbar^2}{c^2 r_t^6} = \frac{m_{e-}c^2}{r_{e-}^3} \longrightarrow \frac{\alpha\hbar c}{2} - \frac{3\pi G\hbar^2 r_{e-}^4}{c^2 r_t^6} - m_{e-}c^2 r_{e-} = 0$$
(A.2)

$$-\frac{\alpha\hbar c}{2r_{e+}^4} + \frac{3\pi G\hbar^2}{c^2 r_t^6} = \frac{m_{e+}c^2}{r_{e+}^3} \longrightarrow -\frac{\alpha\hbar c}{2} + \frac{3\pi G\hbar^2 r_{e+}^4}{c^2 r_t^6} - m_{e+}c^2 r_{e+} = 0$$
(A.3)

$$\begin{split} & \text{C1:}=N[(9.10938356000000000000 \times 10^{-31})((2.99792458000000000000 \times 10^8)^2), 22];\\ & N\,[\text{Solve}\,[A-B*r_{e-}^4/(r_t^6)-\text{C1}*r_{e-}==0,r_{e-}], 22]\,\,//\text{Last}\\ & N\,[\text{Solve}\,[-A+B*r_{e+}^4/(r_t^6)-\text{C1}*r_{e+}==0,r_{e+}], 22]\,\,//\text{Last}\\ & \{r_{e-}\to 8.2143011998060519083\times 10^{-34}\}\\ & \{r_{e+}\to 8.2143011998060519107\times 10^{-34}\} \end{split}$$

# Calculation of Radius $r_{\mu}$ of Muon and Anti-Muon

$$\frac{\alpha\hbar c}{2r_{\mu-}^4} - \frac{3\pi G\hbar^2}{c^2 r_t^6} = \frac{m_{\mu-}c^2}{r_{\mu-}^3} \longrightarrow \frac{\alpha\hbar c}{2} - \frac{3\pi G\hbar^2 r_{\mu-}^4}{c^2 r_t^6} - m_{\mu-}c^2 r_{\mu-} = 0$$
(A.4)

$$-\frac{\alpha\hbar c}{2r_{\mu+}^4} + \frac{3\pi G\hbar^2}{c^2 r_t^6} = \frac{m_{\mu+}c^2}{r_{\mu+}^3} \longrightarrow -\frac{\alpha\hbar c}{2} + \frac{3\pi G\hbar^2 r_{\mu+}^4}{c^2 r_t^6} - m_{\mu+}c^2 r_{\mu+} = 0$$
(A.5)

#### Calculation of Radius $r_{\tau}$ of Tauon and Anti-Tauon

$$\frac{\alpha\hbar c}{2r_{\tau-}^4} - \frac{3\pi G\hbar^2}{c^2 r_t^6} = \frac{m_{\tau-}c^2}{r_{\tau-}^3} \longrightarrow \frac{\alpha\hbar c}{2} - \frac{3\pi G\hbar^2 r_{\tau-}^4}{c^2 r_t^6} - m_{\tau-}c^2 r_{\tau-} = 0$$
(A.6)

$$-\frac{\alpha\hbar c}{2r_{\tau+}^4} + \frac{3\pi G\hbar^2}{c^2 r_t^6} = \frac{m_{\tau+}c^2}{r_{\tau+}^3} \longrightarrow -\frac{\alpha\hbar c}{2} + \frac{3\pi G\hbar^2 r_{\tau+}^4}{c^2 r_t^6} - m_{\tau+}c^2 r_{\tau+} = 0$$
(A.7)

 $\begin{aligned} \text{C3:}=& N[(3.1674700000000000000 \times 10^{-27})((2.99792458000000000000 \times 10^8)^2), 22];\\ & N \left[ \text{Solve} \left[ A - B * r_{\tau^-}^4 / \left( r_t^6 \right) - \text{C3} * r_{\tau^-} == 0, r_{\tau^-} \right], 22 \right] / \text{Last} \\ & N \left[ \text{Solve} \left[ -A + B * r_{\tau^+}^4 / \left( r_t^6 \right) - \text{C3} * r_{\tau^+} == 0, r_{\tau^+} \right], 22 \right] / \text{Last} \\ & \left\{ r_{\tau^-} \to 8.2143011998060505218 \times 10^{-34} \right\} \\ & \left\{ r_{\tau^+} \to 8.2143011998060532972 \times 10^{-34} \right\} \end{aligned}$ 

## Calculation of the Cancellation Radius for Quarks using Formula (41):

$$\frac{2\alpha\hbar c}{9} + 4\alpha_s\hbar c - \frac{3\pi G\hbar^2}{c^2 r_{tq}^2} = \left(\frac{4\alpha\hbar c}{2} + 36\alpha_s\hbar c\right)r_{tq}^2 - \frac{(9)3\pi G\hbar^2}{c^2} = 0$$
(A.8)

$$\begin{split} \mathbf{D}:=& N[(36)(1/10)(1.054571800000000000000000000 \times 10^{-34})(2.9979245800000000000 \times 10^8), 22]; \\ & N\left[\mathrm{Solve}[((4)A+D)*r_{tq}^2-(9)B==0, r_{tq}], 22\right] \; //\mathrm{Last} \end{split}$$

 ${r_{tq} \rightarrow 7.8294227049438663835 \times 10^{-35}}$ 

# Calculation of Radius $r_{qt}$ of Top Quark:

$$\frac{2\alpha\hbar c}{9r_{qt}^4} + \frac{4\alpha_s\hbar c}{r_{qt}^4} - \frac{3\pi G\hbar^2}{c^2 r_{tq}^6} = \frac{m_t c^2}{r_{qt}^3} \longrightarrow \frac{4\alpha\hbar c}{2} + 36\alpha_s\hbar c - \frac{(9)3\pi G\hbar^2 r_{qt}^4}{c^2 r_{tq}^6} - 9\,m_t c^2 r_{qt} = 0 \tag{A.9}$$

$$\begin{split} & \text{E:=}N[9\,(3.087700000000000000 \times 10^{-25})((2.9979245800000000000 \times 10^8)^2), 22]; \\ & N[\text{Solve}[(4)A + D - (9)B * r_{qt}^4 / (r_{tq}^6) - \text{E} * r_{qt} == 0, r_{qt}], 22] \, //\text{Last} \\ & \{r_{qt} \to 7.8294227049438660486 \times 10^{-35}\} \end{split}$$

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